Theoretical Summary

- VaR-based capital charges require VaR to be calculated for a 10 business-day holding period
- In order to calculate VaR based on 10-day returns, there are two approaches both of which have limitations as noted below

    1. Overlapping returns – would result in understated volatility and hence VaR as discussed by *Sun, et. Al.*[1]
    2. Non-overlapping returns – would require unrealistic amounts of data (e.g. to have 1,000 days of simulated P&L, this would require 40+ years of data. Using 4 years of data (100 data points) would result in an unstable VaR statistic.

- Due to limited data availability, MS currently calculates this VaR by scaling 1-day VaR calculation by $\sqrt{10}$; i.e.

$$VaR_{10-Day} \approx VaR_{1-Day} * \sqrt{10}$$

[1]Sun, Nelken, Han, and Guo. *Error of VaR by overlapping intervals.* Asia Risk, April 2009

Theoretical Summary

Appropriateness of the $\sqrt{10} * 1 - Day\ VaR$ scaling for 10-Day VaR Approximation

- For VaR purposes, the following are equivalent if H will be constant for different quantiles and different holding periods (K).

$$VaR(K * x) = K^H[VaR(x)]$$

- In this case, K = 10, and H will be 0.5 if the 10-day VaR is equivalent 1-day VaR scaled by $\sqrt{10}$
- The majority of coefficients are close to 0.5 at 1%tile and 99%tile, indicating that the 10-day VaR is sufficiently approximated by the scaled 1-day VaR.

Historical VaR Hurst Exponents

	EQ	FX	IR	CM	Credit
1%tile	0.4461	0.5040	0.4612	0.4326	0.4191
5%tile	0.4648	0.5184	0.5092	0.4745	0.4386
10%tile	0.4511	0.5150	0.5288	0.5007	0.4460
90%tile	0.5025	0.5090	0.5313	0.5113	0.4329
95%tile	0.4769	0.4808	0.5114	0.4971	0.4369
99%tile	0.4112	0.4489	0.4595	0.4439	0.4465

Definition: A real-valued process $\left(X(t)\right)_{t\in\mathbb{R}}$ is self-similar with index $H > 0$ (H – ss) if for all $a > 0$, the finite-dimensional distributions of $\left(X(at)\right)_{t\in\mathbb{R}}$ are identical to the finite dimensional distributions of $(a^H X(t))_{t\in\mathbb{R}}$, i.e., if for any $a > 0$

$$\left(X(at)\right)_{t\in\mathbb{R}} =^* (a^H X(t))_{t\in\mathbb{R}}$$

- "=" in this instance means equivalently distributed
- EQ – SPX; FX – GBP/USD; IR – 5Yr USD Swap Rate; CM – WTI; Credit – Moody's Baa Spread

H = 'Hurst Coefficient' (a measure of the long-term memory of a time series)